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Small scale coherent vortex generation in drift wave-zonal flow turbulence

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We present a paradigm for the generation of small scale coherent vortex (SSCV) in drift wave-zonal flow (DW-ZF) turbulence. We demonstrate that phases of DWs can couple coherently, mediated by the ZF shearing. A SSCV is formed when the phases of the DWs are "attracted" to form a stable "phase cluster." We show that the ZF shearing induces asymmetry between "attractive" and "repulsive" phase couplings, so that a net attractive phase coupling results. The turbulent DWs will (partially)synchronize into a stable SSCV at locations, where the attractive phase coupling induced by the ZF shearing exceeds the "detuning" effects by the DW dispersion and random phase scattering. We also discuss the "self-binding" effect of the newly formed SSCV.
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I. INTRODUCTION

Small scale coherent vortex is ubiquitous in drift wave-zonal flow (DW-ZF) turbulence. A SSCV is an ordered physical entity emerging from the disordered DW turbulence. In reality, SSCVs exhibit as intermittency of various turbulent flux, e.g., particle flux and heat flux in the edge of magnetically confined plasmas.[1,2] The conventional closure models for turbulence hierarchy equations cannot reproduce all aspects of the SSCV. A limitation of standard approaches is that their use of the random phase approximation, which explicitly fails to account for coherent interaction among the phases of the DWs. Since the SSCV has no clear scale separation from the incoherent DW fluctuations, the use of perturbative analysis for SSCV formation is strongly limited. Numerical simulations in fluid turbulence showed that the generation of SSCV largely follows the shearing by a large scale flow.[3,4] In other words, non-local interaction in wavenumber space plays a crucial role in SSCV generation.

In this work, we propose a new mechanism to SSCV formation—ZF-induced phase synchronization of DWs. Phase synchronization is a useful concept for describing cooperative phenomena (e.g., pattern formation) in multi-degree-of-freedom systems.[5] The classical Kuramoto phase oscillator model[6] proved that a secondary oscillator would emerge once the strength of phase coupling between the elementary oscillators exceeds a critical value. It is not a surprise that, in the quasi-steady DW-ZF turbulence, the dynamical behavior of each DW is governed by its phase evolution. Nonlinear interactions among different DWs are, in turn, determined by their phase couplings. In this work, we found the phase evolution of each DW obeys a Kuramoto equation with a net "attractive" phase coupling induced by ZF shearing, so that a stable "phase cluster" is formed via super-critical bifurcation. Within the phase cluster, the trapped DWs coherently couple with each other—a SSCV emerges. We also argue that the newly formed SSCV may exhibit "self-binding" effect. Our new approach is non-perturbative and so is more broadly applicable.

II. THE PHASE EVOLUTION EQUATION OF THE DW-OSCILLATOR

We develop the theory in the context of the Charney-Hasegawa-Mima (CHM) turbulence system.[7,8] This system is a fundamental model describing linear and nonlinear dynamics in geostrophic- and magnetically confined plasma turbulence. The Fourier representation of the CHM equation is

$$\frac{\partial}{\partial t}\phi_k+i\frac{\omega_{*,e}}{1+\rho_s^2k^2}\phi_k+\underbrace{\sum_{\vec{k'}+\vec{k''}=\vec{k}}\rho_s^2\frac{\vec{k'}\times\vec{k''}\cdot\hat{z}}{1+\rho_s^2k^2}(k'^2-k''^2)\phi_{k'}\phi_{k''}}_{F_{NL}}=0, \tag{1}$$

where ϕ_k is the velocity stream function (it is also electrostatic potential for the plasma application). The coordinate framework we used is sketched in Fig. 1. Without nonlinear interaction, each DW oscillates at its linear frequency, $\frac{\omega_{*,e}}{1+\rho_s^2k^2}$, with $\omega_{*,e}$ the electron diamagnetic frequency and ρ_s the ion-

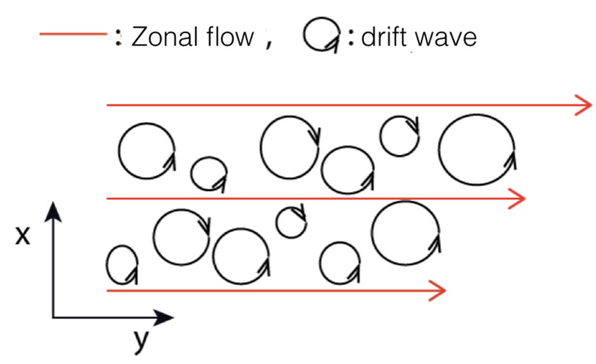

FIG. 1. Set up of the analysis. The $\hat{z}$ direction is perpendicular to the (x, y) plane.

[a)]Author to whom correspondence should be addressed. Electronic mail: guozhipku@gmail.com sound Larmor radius. The nonlinear interaction F_{NL} can be discomposed into a long-short interaction piece(F_{LS}) and a short-short(F_{SS}) piece

$$F_{NL} = F_{LS} + F_{SS}, \tag{2}$$

with

$$F_{LS} = \sum_{\substack{\vec{k}'+\vec{q}=\vec{k},\\ |q|\ll|k'|\sim|k|}} \rho_s^2 \frac{\vec{k}'\times\vec{q}\cdot\hat{z}}{1+\rho_s^2k^2}\left(k'^2-q^2\right)\phi_{k'}\phi_q, \tag{3}$$

$$F_{SS} = \sum_{\substack{\vec{k}'+\vec{k}''=\vec{k},\\ |k|\sim|k'|\sim|k''|}} \rho_s^2 \frac{\vec{k}'\times\vec{k}''\cdot\hat{z}}{1+\rho_s^2k^2}\left(k'^2-k''^2\right)\phi_{k'}\phi_{k''}, \tag{4}$$

where $\vec{q} = q\hat{x}$ is the mode number of the ZF(ϕ_q), mediating mode coupling between ϕ_k and $\phi_{k'}$. ϕ_k can be rewritten as $\phi_k = |\phi_k|e^{iS}$ with $S = S(x, t, k)$ is the Eikonal phase. Where the temporal behavior of the phase in k–space is concerned, we simply set its spatial dependence in a linear form, i.e.,

$$S = \theta_k(t) + \vec{k}\cdot\vec{x}. \tag{5}$$

So that θ_k is independent of spatial variable, and we have $\partial\theta_k/\partial t = d\theta_k/dt$. F_{LS} can then be written as

$$F_{LS} = \sum_{\vec{k}'+\vec{q}=\vec{k},|q|\ll|k'|\sim|k|} \frac{q^2k_y\rho_s^2k^2}{1+\rho_s^2k^2}|\phi_q|\frac{\partial|\phi_k|}{\partial k_x}e^{i(\theta_q+\theta_{k'})+i\vec{k}\cdot\vec{x}}. \tag{6}$$

Here, $|\phi_{k'}|$ has been approximated by Taylor expansion of k' about k, $|\phi_{k'}| \simeq -q\frac{\partial}{\partial k_x}|\phi_k|$. In the absence of phase synchronization, the phases of the DWs are randomly distributed, so that the short-short interaction obeys a random phase approximation and F_{SS} can be modeled as noise, which is relevant to turbulent phase mixing.[9] Really, coherent phase coupling comes via long-short interaction, so long-short interaction does the synchronization, while short-short interaction drives noise.

Regarding the phase dynamics of the DW modes, we make the following assumption: the turbulence is in a nonlinearly saturated quasi-steady state, i.e., $\partial_t\ln|\phi_k| \ll \partial_t\ln|\theta_k|$. The phase of the straining field changes slowly, so that we simply choose θ_q as a constant and $\theta_q = 0$ without loss of generality. According to Eq. (1), the evolution of θ_k is derived as

$$\frac{d}{dt}\theta_k = -\omega_k + \sum_{k'} C_kV'_{ZF,q}\sin(\theta_{k'}-\theta_k) + s_k, \tag{7}$$

where $\omega_k = \frac{\omega_{*,e}}{1+\rho_s^2k^2}$ and $V'_{ZF,q} = -q^2|\phi_q|$ is the shear-rate of the ZF. s_k accounts for random phase scattering induced by the short-short interaction, and the coupling coefficient is

$$C_k \equiv \frac{k_y\rho_s^2k^2}{1+\rho_s^2k^2}\frac{1}{2|\phi_k|^2}\frac{\partial|\phi_k|^2}{\partial k_x}. \tag{8}$$

Since the structure of the turbulence spectrum will be modulated by the ZF shearing, one can expect that the averaged C_k is also a function of the ZF shear. Eq. (7) describes a generalized Kuramoto system—the most representative model of synchronization phenomena in populations of coupled oscillators.[10] In deriving Eq. (7), we assumed all the phases of the large scale field take the same value (i.e., $\theta_q = 0$). A more general case is that different ϕ_q has different θ_q, which is equivalent to adding a random phase to phase coupling term of Eq. (7). The randomness of θ_q has an effect of reducing the coherence of θ_ks. The inclusion of the distribution of θ_q will not impact our conclusion qualitatively but increase the complicity of analytical analysis significantly,[11] so we ignore this effect in our current work. If $C_kV'_{ZF,q} > 0$, θ_k and $\theta_{k'}$ are "attractively" coupled, and if $C_kV'_{ZF,q} < 0$, they are "repulsively" coupled. A net attractive coupling could facilitate the formation of a phase cluster, and make the DWs self organize into a SSCV. It has been shown[12,13] that the dynamical behavior of Eq. (7) is equivalently captured by the following reduced equation:

$$\frac{d}{dt}\theta_k = -\omega_k + \bar{C}V'_{ZF}\sum_{k'}\sin(\theta_{k'}-\theta_k) + s_k, \tag{9}$$

where $\bar{C}$ is the averaged phase coupling coefficient, weighted by the distribution function of the DWs. $V'_{ZF} = \sum_q V'_{ZF,q}$ is the total shear-rate. Without long-short interaction, each test phase experiences linear oscillation (the 1st term on the RHS of Eq. (9)) and nonlinear random scattering (the 3rd term on the RHS of Eq. (9)), so the conventional mixing length theory is applicable. For DW-ZF turbulence, the population of the DWs is proportional to its potential enstrophy intensity,[9] $\Omega_k = \frac{1}{2}(1+\rho_s^2k^2)^2|\phi_k|^2$. Then, we have

$$\bar{C} = \frac{\sum_k \frac{k_y\rho_s^2k^2}{1+\rho_s^2k^2}\frac{1}{2|\phi_k|^2}\frac{\partial|\phi_k|^2}{\partial k_x}\Omega_k}{\sum_k\Omega_k}. \tag{10}$$

Since the ZF can modulate the radial wavenumber of the DW, Ω_k is also a function of V'_{ZF}. To calculate the radial wavenumber variation by ZF shearing, we employ the ray equation, $\partial_tk_x = -k_yV'_{ZF}$. Then, we obtain $\Delta k_x = -k_yV'_{ZF}\Delta t$. Since the Δt can not exceed the correlation time ($\Delta\omega^{-1}$) of the DW turbulence, we simply set $\Delta t \simeq \Delta\omega_k^{-1}$ and thus we have $\Delta k_x \simeq -k_yV'_{ZF}/\Delta\omega_k$. Because of the slow change of the ZF, Ω_k approximately undergoes an adiabatic variation, so that we have $\Omega_k = \Omega_0(k_x-\Delta k_x, k_y) \simeq \Omega_{0,k} + \Delta k_x\partial_{k_x}\Omega_{0,k}$ with $\Omega_{0,k}$ the potential enstrophy in the absence of ZF. Substituting it into Eq. (10) yields

$$\bar{C} \simeq \frac{1}{\Omega}\sum_k\frac{k_y^2\rho_s^2k^2}{1+\rho_s^2k^2}\frac{1}{2\Delta\omega_k|\phi_k|^2}\frac{\partial|\phi_k|^2}{\partial k_x}\frac{\partial\Omega_{0,k}}{\partial k_x}V'_{ZF} \equiv \beta V'_{ZF}, \tag{11}$$

where $\Omega = \sum_k\Omega_k$ is the total potential enstrophy intensity. β scales with DW turbulence decorrelation time, $\beta \sim \Delta\,\omega^{-1}$. For a normal profile of DW spectrum with $\partial_{k_x}|\phi_k|^2\partial_{k_x}\Omega_k > 0$, we have

$$\bar{C}V'_{ZF} = \beta V'^2_{ZF} > 0. \tag{12}$$

As a result, the ZF shearing induces a net positive phase coupling—a necessary condition of forming a phase cluster/SSCV. Physically, because of the coherent phase-coupling among the DWs (induced by the ZF shearing), the ray trajectories of the DWs are nonlinearly modulated. As a result, some wave-packets approach to each other, are trapped, and then form a SSCV.

III. PHASE SYNCHRONIZATION OF THE DW-OSCILLATORS

An illuminating way to quantitatively describe the collective behavior of θ_k is to introduce an order parameter[6] $re^{i\Theta} = 1/N\sum_k e^{i\theta_k}$ with N the total number of the DW-packets. r is a real number charactering the coherence of the DW phases: $r=0$ ($r=1$) for completely incoherent(coherent) state and $0<r<1$ for partially coherent state. Θ is the average phase describing the collective oscillation of the DWs (Fig. 2). Without loss of generality, we can always set $\Theta=0$ as measured in a certain rotating frame.[10] Taking the continuous limit (from now on, we remove the subscript k) yields

$$r = \int\int e^{i\theta}\rho(\theta,\omega,t)g(\omega)d\theta d\omega. \tag{13}$$

Here, $\rho(\theta,\ \omega,\ t)$ is the distribution of θ at fixed frequency ω, and $g(\omega)$ is the distribution of ω. $\rho(\theta,\ \omega)$ and $g(\omega)$ satisfy the normalization conditions: $\int\rho(\theta,\omega)d\theta = 1$ and $\int g(\omega)d\omega = 1$. Employing Eq. (13), Eq. (9) can be rewritten as[6]

$$\frac{d}{dt}\theta = -\omega + \beta V_{ZF}'^2 r\sin\theta + s. \tag{14}$$

Eq. (14) is a reduced governing equation of the evolution of the DW phase. The phase dynamic of each DW-oscillator is determined by the competition of the detuning effects (the linear wave dispersion and the noise effect) and the pining effect (ZF shearing). For the purpose of illustrating the essential physics, we make a detail discussion of the weak noise scenario. The weak noise scenario corresponds to a region with strong ZF shearing. Because the DW turbulence has a spatial distribution and the turbulence intensity can be large in other locations, and so, on the average, the turbulence does not collapse. Then, Eq. (14) exactly corresponds to the original Kuramoto model

$$\frac{d}{dt}\theta \simeq -\omega + \beta V_{ZF}'^2 r\sin\theta. \tag{15}$$

Solutions of Eq. (15) fall into two groups: phase-locked ones ($|\omega| < \beta V_{ZF}'^2$) and phase-slipping ones ($|\omega| > \beta V_{ZF}'^2$). For the phase-locked solution, θ is attracted to a mean value and

$$\theta = \arcsin\frac{\omega}{\beta V_{ZF}'^2 r}. \tag{16}$$

For the phase-slipping ones, each θ moves in a nonuniform way (Fig. 2). The ZF shearing plays a crucial role in determining the value of r. For example, in locations with strong flow shearing, the detuning effects associated with the linear frequency dispersion is negligible, so all the DW oscillators are attracted to the same phase and evolve into a completely synchronized state, $r\to 1$. Here the strong shearing can be sustained via turbulence spreading. It is worth to point out that the strong shearing scenario does not contradict to turbulence suppression theory,[14] because the ZF has a spatial distribution and its shear rate can be below the threshold in other locations. Thus, on the average, the DW turbulence does not collapse. In the weak noise scenario, a critical shear-rate for the transition from incoherent state to coherent state (formation of a SSCV) can be derived by using the self-consistency condition between Eqs. (13) and (15) and it is[10]

$$V_{ZF,c}'^2 = \frac{2}{\beta\pi g(\bar{\omega})}, \tag{17}$$

where $\bar{\omega}$ is the centroid frequency of the DW frequency distribution function $g(\omega)$. For a homogeneous distribution, $g(\bar{\omega}) = \frac{1}{\bar{\omega}}$, one has

$$V_{ZF,c}'^2 = \frac{2}{\beta\pi}\bar{\omega} \simeq \frac{2}{\beta\pi}\Delta\omega\bar{\omega}. \tag{18}$$

Therefore, the critical ZF shear-rate for the generation of a SSCV is proportional to the geometry average of $\Delta\omega$ and $\bar{\omega}$, i.e.,

$$V_{ZF,c}' \sim \sqrt{\Delta\omega\bar{\omega}}. \tag{19}$$

Once $V_{ZF}'^2 > V_{ZF,c}'^2$, a phase cluster (i.e., a SSCV) will form (Fig. 2). This scaling is consistent the scaling obtained by requiring the pinning effect exceeds the detuning effect in Eq. (15). It has also been proved that the newly formed organized state is stable.[15] Near the transition point, r takes the form

$$r \simeq \frac{4}{\beta^2 V_{ZF,c}'^4}\sqrt{\frac{\beta}{-\pi g(\bar{\omega})''}}\sqrt{V_{ZF}'^2 - V_{ZF,c}'^2}, \tag{20}$$

where $-g(\bar{\omega})'' > 0$ for generic case, e.g., the Gaussian distribution. So, near the critical point ($r\to 0+$) $\partial r/\partial V_{ZF}'^2 \sim (V_{ZF}'^2 - V_{ZF,c}'^2)^{-1/2} \to +\infty$, and the generation of a SSCV is a supercritical bifurcation process.

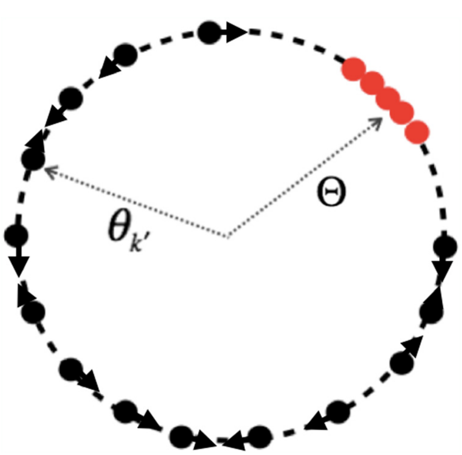

FIG. 2. Sketch of a phase cluster in a phase ring: black points—phase slips with $\theta_{k'}$ the phase of the incoherent DW, red points—phase locked with Θ the phase of the coherent DWs.

The stream function of the newly formed SSCV can be written as $\phi_{SSCV} = \sum |\phi_k| \exp(i\vec{k} \cdot \vec{x})$ with the sumation being over all the phase locked DWs. The amplitude of the SSCV is strongest at the center ($x = 0$). Away from $x = 0$, $\vec{k} \cdot \vec{x}$ will induce phase differences among different modes, so that the synchronized DWs start to loss coherence in space. This process limits the spatial extent (l_{SSCV}) of the SSCV, and it can be estimated as $l_{SSCV} \simeq \Delta k^{-1}$ with Δk^{-1} the width of the wavenumber spectrum of the synchronized DWs. An interesting deduction is that the larger the population of the trapped DWs, the smaller the spatial extent of the SSCV will be. The reason is that more DWs usually mean a broader wavenumber spectrum, so that Δk^{-1} is smaller.

IV. SELF-BINDING EFFECT

Along with the formation of a phase cluster, the short-short phase coupling among the to-be-synchronized DWs changes from incoherent coupling to coherent one. Therefore, F_{SS} is not a pure noise any more, but exhibits coherence. The newly generated coherent coupling adds a term

$$\mathrm{Im}\left(\frac{F_{SS}}{|\phi_k|}\right) = \alpha r \sin\theta, \tag{21}$$

to the phase evolution equation of the trapped DW (Eq. (15)). The coupling coefficient is

$$\alpha = \sum_{|k| \sim |k'| \sim |k''|} \rho_s^2 \frac{\vec{k}' \times \vec{k}'' \cdot \hat{z}}{1 + \rho_s^2 k^2} (k'^2 - k''^2) \frac{|\phi_{k'}||\phi_{k''}|}{|\phi_k|}$$

Generally, α is nonzero. If it is positive, the short-short interaction among the trapped DWs exerts another "pinning" force on themselves—the self-binding effect. Then that the SSCV may be sustained even if the ZF shearing is reduced to below its threshold. Physically, this is equivalent to forming a transport barrier by the SSCV itself. If the coefficient is negative, the short-short interaction tends to reduce the effective shear-rate of the ZF, so that a stronger ZF shearing is required to maintain such a SSCV.

V. CONCLUSION

A new mechanism, based on phase synchronization, of SSCV is proposed. The SSCV corresponds to a stable phase cluster of the DWs. In the quasi-steady state, it is shown that ZF shearing induces a net positive couplings among the short scale DWs and, so that, facilitates the formation of the phase cluster. Our theory provides a new angle to look at various SSCV phenomena in turbulence system. For future research, it is important to: (1) explore the feedback of the SSCV on the ZF and (2) extend the current 0D theory to 1D scenario, i.e., including phase-phase coupling in configuration space.

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